$$y = ax^2 + bx + c$$

quadratic formula.

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a} \; ; \quad \boxed{\Delta = b^2 - 4ac}$$

$$x_2 = \frac{-b-\sqrt{\Delta}}{2a} \; ; \quad -||-$$

Proof for:

$$a(x-x_1)(x-x_2) = ax^2+bx+c$$

Root based expression of f(x)

$$a\left(x - \frac{-b+\sqrt{\Delta}}{2a}\right)\left(x - \frac{-b-\sqrt{\Delta}}{2a}\right) =$$

$$= a\left[x^2 - \frac{-b-\sqrt{\Delta}}{2a}x - \frac{-b+\sqrt{\Delta}}{2a}x + \frac{(-b+\sqrt{\Delta})(-b-\sqrt{\Delta})}{4a^2}\right]$$

$$= a\left[x^2 - \frac{(-b-\sqrt{\Delta})x + (-b+\sqrt{\Delta})x}{2a} + \frac{(-b)^2 - (\sqrt{\Delta})^2}{4a^2}\right]$$

$$= a\left[x^2 - \frac{x(-b-\cancel{\sqrt{\Delta}}-b+\cancel{\sqrt{\Delta}})}{2a} + \frac{b^2-\Delta}{4a^2}\right]$$

$$= a\left[x^2 - \frac{x(-\cancel{2}b)}{\cancel{2}a} + \frac{b^2-(b^2-4ac)}{4a^2}\right] =$$

$$= a\left[x^2 + \frac{b}{a}x + \frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a^2}\right] =$$

$$= a\left[x^2 + \frac{b}{a}x + \frac{\cancel{4}\cdot\cancel{a}\cdot c}{\cancel{4}\cdot\cancel{a}\cdot a}\right] = ax^2 + \cancel{a}\cdot\frac{b}{\cancel{a}}x + \cancel{a}\cdot\frac{c}{\cancel{a}}$$

$$= ax^2 + bx + c \quad \leftarrow \text{proven}$$

∴ if $f(x)$ is known expressing $f(x)$ based on roots is easy: $a(x-x_1)(x-x_2)$

The same applies to $S = x_1 + x_2$

$p = x_1 \cdot x_2$.

if $f(x) = ax^2 + bx + c$

$\Rightarrow f(x) = a(x^2 - Sx + p)$

$\boxed{\text{the same } a!}$

PROOF:

$f(x) = a(x - x_1)(x - x_2) =$

root based expr.

$= a(x^2 - x \cdot x_2 - x \cdot x_1 + \underbrace{x_1 \cdot x_2}_{\text{"}P\text{"}})$

$= a(x^2 - x\underbrace{(x_2 + x_1)}_{S} + p)$

$= a(x^2 - Sx + p)$

∴ if the coefficients of $f(x)$ are given or rather just the 1st coefficient: a and the roots (either individually or as sum & product) then the alternate form exists

$a(x^2 - Sx + p)$