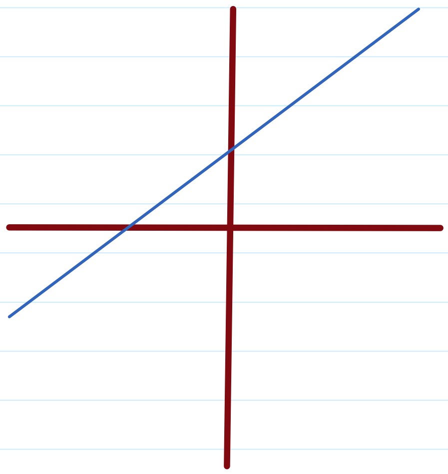
Distance Between Parallel lines

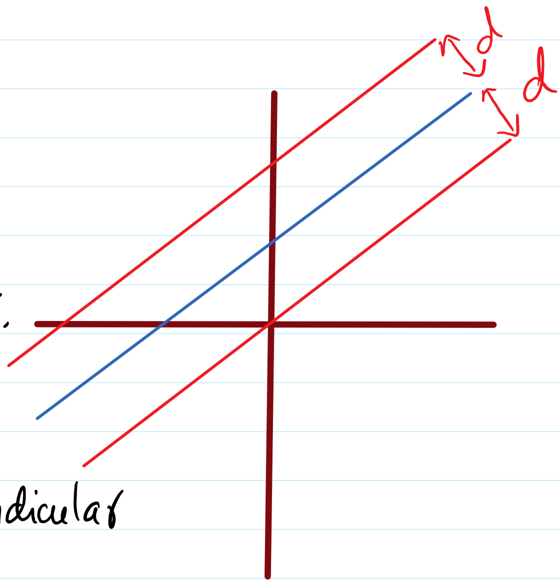
Saturday, December 2, 2017 7:37 PM

What is a general form to find the formula of a parallel line at a given distance d?



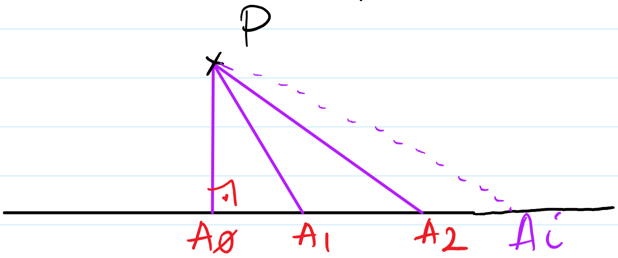
$y_1 = mx + b_1$ ← the general form of a given line

Given a distance "d" there should be 2 lines parallel to the blue one, at that distance



Remember that the distance between parallel lines is given by the ~~the~~ perpendicular.

The shortest distance between a point and a line is the perpendicular



the further from A the longer the line PA_i gets.

Here are a few things we know given a formula: $y_1 = mx + b_1$,

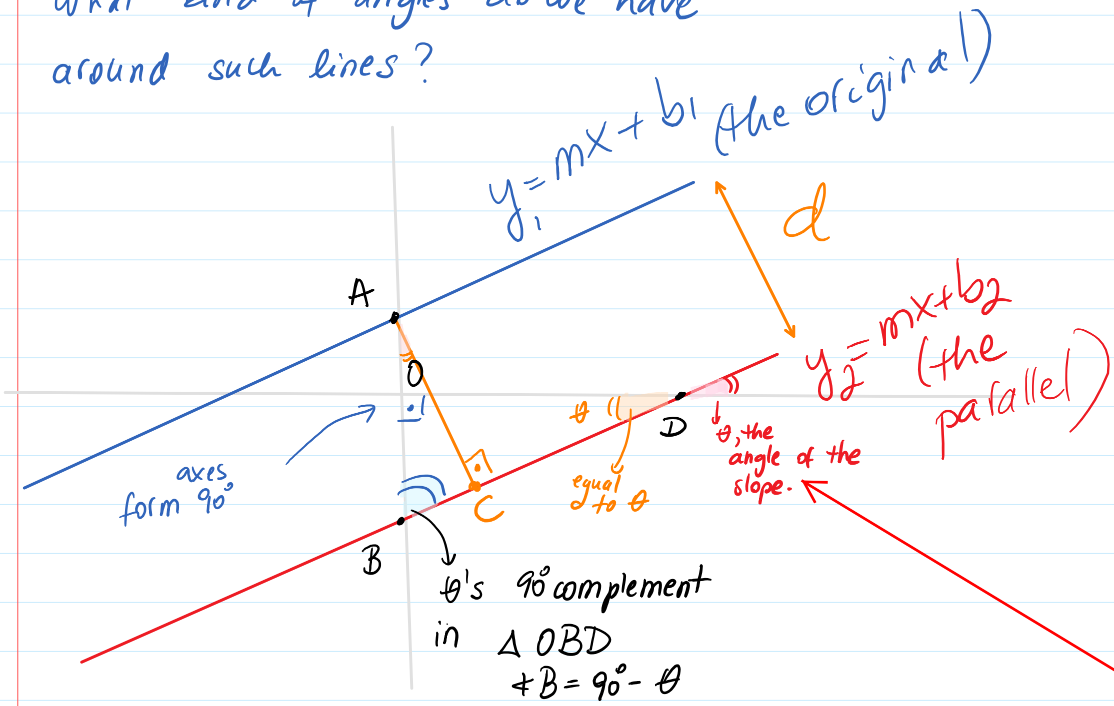
any parallel line y_2 would have to be:

$$y_2 = mx + b_2$$

i.e. their slopes are identical.

The problem of finding the line y_2 at a given distance d, becomes finding its b_2 value (its y-intercept).

What kind of angles do we have around such lines?



Thus in ΔABC: $\measuredangle C = 90^\circ$

$\measuredangle B = 90^\circ - \theta$

$\measuredangle A = 90^\circ - (90^\circ - \theta) = \theta$

This is important b/c:

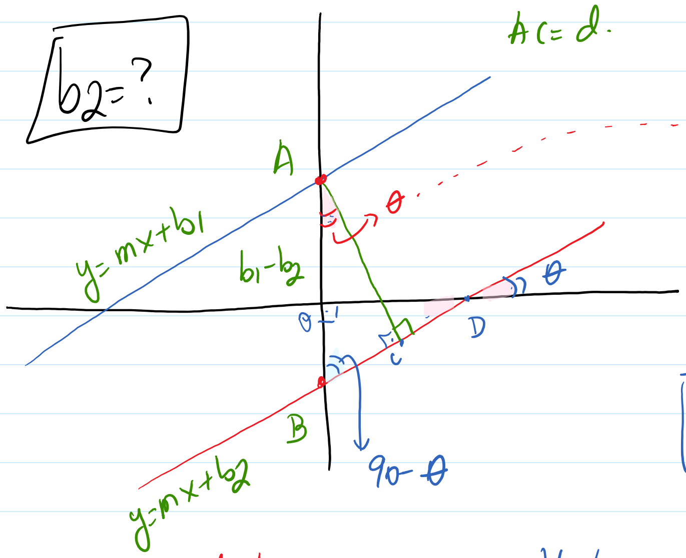
θ is the angle of the slope

$\Rightarrow \boxed{\tan\theta = m}$

Back to our problem, what is the y-intercept?

$AC = d.$

$\boxed{b_2 = ?}$



in $\triangle ABC$ $\rightarrow$ $\tan\theta = \frac{BC}{d}$

$m = \frac{BC}{d}$

$\Downarrow$

$\boxed{BC = m \cdot d}$

we can now replace tan(θ) with m, and furthermore with its trigonometry: BC/d

Values considered known:

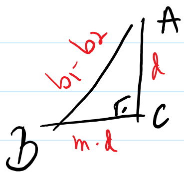
m

b_1

d

Values to find:

b_2



$$AB^2 = BC^2 + AC^2.$$

$$(b_1 - b_2)^2 = (md)^2 + d^2$$

$$(b_1 - b_2)^2 = m^2d^2 + d^2$$

$$(b_1 - b_2) = \pm\sqrt{d^2(m^2+1)} = \pm\sqrt{d^2}\cdot\sqrt{m^2+1}$$

$$= \pm d\sqrt{m^2+1}$$

$$b_1 - b_2 = \pm d\sqrt{m^2+1}$$

$$b_1 = \pm d\sqrt{m^2+1} + b_2$$

$$b_1 \mp d\sqrt{m^2+1} = b_2$$

$$\therefore \boxed{b_2 = b_1 \pm d\sqrt{m^2+1}}$$

there are two solutions: ±
b/c one parallel is above, the other below the original line.

Conclusion

For a line $y = mx + b_1$, the two parallel lines found at a distance d from this line are:

$$y = mx + b_1 \pm d\sqrt{m^2+1}$$

$\rightarrow /\!/.$